

Toolbox 6

THERMODYNAMIC AND TRANSPORT PROPERTIES OF MOIST AIR

1. **Definitions.** Moist air is a mixture of *dry air* (which is itself a mixture of a number of gases) and *water vapor*. The water vapor may exist in a saturated or superheated state. Dry air and water vapor form a binary mixture. A mixture of *two substances* requires *three thermodynamic properties* to define its thermodynamic state completely.

Both dry and water vapor can be considered as perfect gases since both exist in the atmosphere at relatively low pressures. Water vapor is present in the atmosphere at a absolutely low partial pressure.

The properties of moist air are called psychrometric properties and the topic which deals with the behavior of moist air is known as psychrometry.

For defining and calculating the relevant psychrometric properties, one can consider a certain volume V of moist air at mixture pressure P and temperature T , containing m_a [kg] of dry air and m_v [kg] of water vapor as shown in Fig. 6.1.

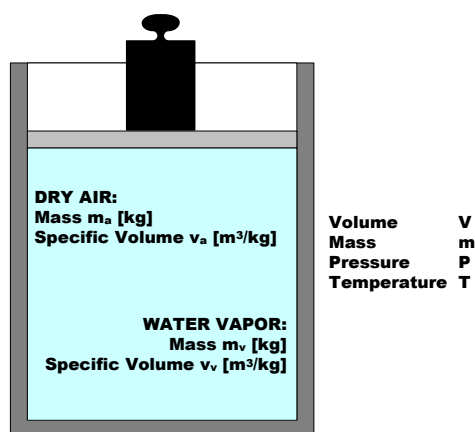


Figure 6.1: A Mixture of Dry Air and Water Vapor (Moist Air)

One can recognize the following regions in Mollier's diagram (Fig. 2):

- A. Unsaturated region
- B. Foggy region

- C. Ice-cold fog region
- D. Ice and fog mixture region

For practical industrial applications the most important region is A. The majority of industrial processes occur in this region and, because of that, only unsaturated moist air will be analyzed.

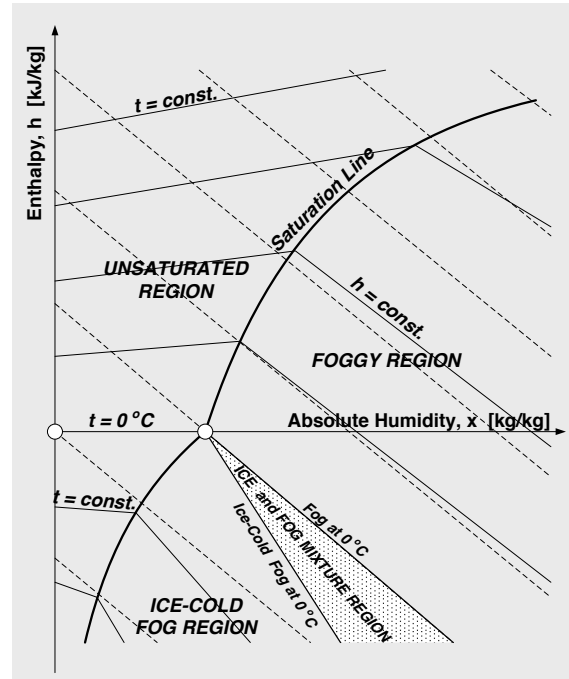


Figure 6.2: Mollier's Diagram for Moist Air at Constant Pressure

2. Dry Bulb Temperature is the temperature of moist air measured by an ordinary thermometer. This term is used only to distinguish it from wet bulb temperature.

3. Absolute Humidity is defined as the ratio of the mass of water vapor to the mass of dry air in a given volume of the mixture. Thus

$$x = \frac{m_v}{m_a} \quad (6.1)$$

where the subscripts a and v refer to **dry air** and **water vapor**, respectively.

Considering both dry air and water vapor as perfect gases it can be obtained that the absolute humidity is as follows:

$$x = 0.622 \cdot \frac{p_v}{p_a} = 0.622 \cdot \frac{p_v}{P - p_v} \quad (6.2)$$

As the absolute humidity is defined as mass of water vapor per mass of *dry air*, the total mass of moist air will be as follows:

$$m = (1 + x) \cdot m_a \quad (6.3)$$

4. The Relative Humidity of the moist air is the ratio of the actual mass of the water vapor in a given volume to that which it would have if it were saturated at the same dry bulb temperature and the same total pressure of the air:

$$RH = \frac{m_v}{m_{v,sat}} \quad (6.4)$$

or, it can be defined as the ratio of partial pressure of water vapor in unsaturated air and the partial pressure of vapor if it is saturated at the same temperature:

$$RH = \frac{p_v}{p_{v,sat}} \quad (6.5)$$

Relative Humidity is frequently expressed as a percentage.

5. **Dew Point Temperature.** If unsaturated moist air is cooled at constant pressure, the mixture will eventually reach the saturation temperature of water vapor corresponding to its partial pressure at the point at which the first drop of dew will be formed, i.e. the water vapor in the mixture will be saturated. This temperature is called the dew point temperature t_{dp} [°C] and it is, therefore, the temperature to which unsaturated moist air must be cooled at constant pressure before condensation of moisture takes place.

6. **Wet Bulb Temperature.** Thermodynamic wet bulb temperature is the temperature at which liquid or solid water, by evaporating into air, may bring the air to saturation adiabatically at the same temperature. Wet bulb temperature is the temperature indicated by a wet bulb psychrometer constructed and used according to specifications.

7. The enthalpy of unsaturated moist air or the moist air on saturation line is obtained by summation of the enthalpies of the constituents. Thus the enthalpy of moist air is equal to the sum of the enthalpies of dry air and associated water vapor:

$$h = h_a + x \cdot h_v \quad (6.6)$$

where:

- h = enthalpy of moist air, [kJ/kg_{dry air}]
- h_a = enthalpy of dry air, [kJ/kg_{dry air}]
- h_v = enthalpy of water vapor, [kJ/kg_{vapor}]
- x = absolute humidity, [kg_{vapor}/kg_{dry air}]

Taking the reference state enthalpy as zero for saturated liquid at 0 °C, and that the latent heat of vaporization at this temperature is 2500.84 kJ/kg, and taking that the specific heat at constant pressure of dry air and water vapor are:

$$c_{p,a} = 1.0029 + 5.4 \cdot 10^{-5} \cdot t \quad [\text{kJ/kg K}] \quad (6.7)$$

$$c_{p,v} = 1.856 + 2.0 \cdot 10^{-4} \cdot t \quad [\text{kJ/kg K}] \quad (6.8)$$

the enthalpy of unsaturated moist air is as follows:

$$h = (1.0029 + 5.4 \cdot 10^{-5} \cdot t) \cdot t + x \cdot [500.84 + (1.856 + 2.0 \cdot 10^{-4} \cdot t) \cdot t] \quad [\text{kJ / kg}_{da}] \quad (6.9)$$

This equation could be used in a range from -40 to 300 °C. The total pressure of the air could be changed from 0.1 to 5 bar. In those ranges of temperature and pressures this equation gives good results for many technical calculations.

8. The specific heat of moist air is as follows:

$$c_p = (1.0029 + 5.4 \cdot 10^{-5} \cdot t) + x \cdot (1.856 + 2.0 \cdot 10^{-4} \cdot t) \quad [\text{kJ/kg}_{\text{da}} \text{ K}] \quad (6.10)$$

9. The density of moist air is defined as follows:

$$\rho = \frac{1+x}{461.56 \cdot (0.62198 + x)} \cdot \frac{P [\text{Pa}]}{t [^\circ\text{C}] + 273.15} \quad [\text{m}^3 / \text{kg}] \quad (6.11)$$

10. The dynamic viscosity of moist air can be calculated by using following equation:

$$\mu = \frac{\mu_A}{1 + \Phi_{AV} \cdot x_m} + \frac{\mu_V}{1 + \frac{\Phi_{VA}}{x_m}} \quad (6.12)$$

Where:

$$\Phi_{AV} = \frac{\left[1 + \left(\frac{\mu_A}{\mu_V} \right)^{0.5} \cdot \left(\frac{m_V}{m_A} \right)^{0.25} \right]^2}{2\sqrt{2} \cdot \left(1 + \frac{m_A}{m_V} \right)^{0.5}} \quad \Phi_{VA} = \frac{\left[1 + \left(\frac{\mu_V}{\mu_A} \right)^{0.5} \cdot \left(\frac{m_A}{m_V} \right)^{0.25} \right]^2}{2\sqrt{2} \cdot \left(1 + \frac{m_V}{m_A} \right)^{0.5}}$$

$$x_m = 1.61 \times x$$

$$m_A = 29 \quad \text{Molecular mass of dry air, [kg/kmol]}$$

$$m_V = 18 \quad \text{Molecular mass of dry air, [kg/kmol]}$$

and

$$\mu_a \cdot 10^6 = 0.40401 + 0.074582 \cdot T - 5.7171 \cdot 10^{-5} \cdot T^2 + 2.9928 \cdot 10^{-8} \cdot T^3 - 6.2524 \cdot 10^{-12} \cdot T^4 \quad [\text{Pa s}] \quad (6.13)$$

$$\mu_v \cdot 10^6 = \frac{\sqrt{\frac{T}{647.27}}}{0.0181583 + 0.0177624 \cdot \left(\frac{647.27}{T} \right) + 0.0105287 \cdot \left(\frac{647.27}{T} \right)^2 - 0.0036744 \cdot \left(\frac{647.27}{T} \right)^3} \quad [\text{Pa s}] \quad (6.14)$$

are the dynamic viscosity of dry air and water vapor under low pressure, respectively. In the last two equations $T [\text{K}] = t [^\circ\text{C}] + 273.15$.

11. The thermal conductivity of moist air is:

$$k = \frac{k_A}{1 + A_{AV} \cdot x_m} + \frac{k_V}{1 + \frac{A_{VA}}{x_m}} \quad (6.15)$$

Where:

$$A_{AV} = 0.25 \cdot \left(+\alpha_{AV}^{0.5} \right) \cdot \beta_{AV} \quad \alpha_{AV} = \frac{\mu_A}{\mu_V} \cdot \left(\frac{m_V}{m_A} \right)^{0.75} \cdot \frac{1 + \frac{S_A}{T}}{1 + \frac{S_V}{T}} \quad \beta_{AV} = \frac{1 + \frac{S_{AV}}{T}}{1 + \frac{S_A}{T}}$$

$$A_{VA} = 0.25 \cdot \left(+\alpha_{VA}^{0.5} \right) \cdot \beta_{VA} \quad \alpha_{VA} = \frac{\mu_V}{\mu_A} \cdot \left(\frac{m_A}{m_V} \right)^{0.75} \cdot \frac{1 + \frac{S_V}{T}}{1 + \frac{S_A}{T}} \quad \beta_{VA} = \frac{1 + \frac{S_{VA}}{T}}{1 + \frac{S_V}{T}}$$

and $S_{AV} = 0.733 \cdot \sqrt{S_A \cdot S_V}$. $S_A = 111$ [K]; $S_V = 961$ [K] Sutherland constants for air and water vapor, respectively.

The thermal conductivity of dry air in the range from -50 to +500 °C and for pressure 1.013 bar is as follows:

$$k_A = 2.43714 \cdot 10^{-2} + 7.83035 \cdot 10^{-5} \cdot t - 1.94021 \cdot 10^{-8} \cdot t^2 + 2.85943 \cdot 10^{-12} \cdot t^3 - 2.61420 \cdot 10^{-14} \cdot t^4 \quad (6.16)$$

and for saturated water vapor in the range from 0 to +220 °C is as follows:

$$k_V = 1.74822 \cdot 10^{-2} + 7.69127 \cdot 10^{-5} \cdot t - 3.23464 \cdot 10^{-7} \cdot t^2 + 2.59524 \cdot 10^{-9} \cdot t^3 - 3.17650 \cdot 10^{-12} \cdot t^4 \quad (6.17)$$

All those parameters of unsaturated moist air and moist air on the saturated line can be calculated by using Software 6.

12. **Prandtl Number.** This number is calculated by using the isobaric specific heat of moist air. The influence of the total pressure of moist air on transport properties is relatively small and is not considered in the presented formulae. This has to be kept in mind when the software is used.

13. **Software.** The determination of the thermodynamic and transport properties of moist air can be done by using *Software 6: Thermodynamic Properties of Moist Air*.

References

- Bosnjakovic, F. (1976) *Nauka o toplini, II dio, IV izdanje, Tehnička knjiga*, Zagreb, Croatia.
 Chysky, J. (1977) *Vláhký vzduch*, SNTL, Praha.
 Häussler, W. (1960) *DAS MOLLIER – ix – DIAGRAMM für feuchte Luft und seine technischen Anwendungen*, Verlag von Theodor Steinkopff, Dresden und Leipzig.